

NTMF036

INTERPRETACE KVANTOVÉ MECHANIKY

Shrnutí 9. přednášky

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Bellovy nerovnosti

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 On the Einstein Podolsky Rosen Paradox
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III.5 ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

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I. Introduction

THE paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality [2]. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty. There have been attempts [3] to show that even without such a separability or locality requirement no "hidden variable" interpretation of quantum mechanics is possible. These attempts have been examined elsewhere [4] and found wanting. Moreover, a hidden variable interpretation of elementary quantum theory [5] has been explicitly constructed. That particular interpretation has indeed a grossly non-local structure. This is characteristic, according to the result to be proved here, of any such theory which reproduces exactly the quantum mechanical predictions.

II. Formulation

With the example advocated by Bohm and Aharonov [6], the EPR argument is the following. Consider a pair of spin one-half particles formed somehow in the singlet spin state and moving freely in opposite directions. Measurements can be made, say by Stern-Gerlach magnets, on selected components of the spins $\vec{\sigma}_1$ and $\vec{\sigma}_2$. If measurement of the component $\vec{\sigma}_1 \cdot \vec{a}$, where \vec{a} is some unit vector, yields the value $+1$ then, according to quantum mechanics, measurement of $\vec{\sigma}_2 \cdot \vec{b}$ must yield the value -1 and vice versa. Now we make the hypothesis (2), and it seems one at least worth considering, that if the two measurements are made at places remote from one another the orientation of one magnet does not influence the result obtained with the other. Since we can predict in advance the result of measuring any chosen component of $\vec{\sigma}_2$, by previously measuring the same component of $\vec{\sigma}_1$, it follows that the result of any such measurement must actually be predetermined. Since the initial quantum mechanical wave function does not determine the result of an individual measurement, this predetermination implies the possibility of a more complete specification of the state.

Let this more complete specification be effected by means of parameters λ . It is a matter of indifference in the following whether λ denotes a single variable or a set, or even a set of functions, and whether the variables are discrete or continuous. However, we write as if λ were a single continuous parameter. The result A of measuring $\vec{\sigma}_1 \cdot \vec{a}$ is then determined by \vec{a} and λ , and the result B of measuring $\vec{\sigma}_2 \cdot \vec{b}$ in the same instance is determined by \vec{b} and λ , and

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BELL

$$A(\vec{a}, \lambda) = \pm 1. \quad (1)$$

Article 2 does not depend on the setting \vec{a} , of the magnet or the expectation value of the product of the two com-

$$\rho(\lambda) A(\vec{a}, \lambda) B(\vec{b}, \lambda) \quad (2)$$

tion value, which for the singlet state is

$$\vec{a} \cdot \vec{b} = -\vec{a} \cdot \vec{b}. \quad (3)$$

Hidden variables fall into two sets, with A dependent on \vec{a} and λ as above, since λ stands for any number of variables unrestricted. In a complete physical theory of the world there would have dynamical significance and laws of motion; these variables at some suitable instant.

Illustration

Before giving it, however, a number of illustrations may

be given. In a variable account of spin measurements on a single pure spin state with polarization denoted by a unit vector $\vec{\lambda}$ with uniform probability distribution over all directions, and take

$$\vec{\lambda} \cdot \vec{a}' = \cos \theta. \quad (4)$$

For a way to be specified, and the sign function is ± 1 or -1 and this leaves the result undetermined when $\lambda \cdot \vec{a}' = 0$, or the special prescriptions for it. Averaging over $\vec{\lambda}$ the

$$\langle A \rangle = 1 - 2\theta/\pi, \quad (5)$$

then that \vec{a}' is obtained from \vec{a} by rotation towards \vec{b}

$$\vec{a}' \cdot \vec{b} = \cos \theta \quad (6)$$

$$\langle A \rangle = \cos \theta \quad (7)$$

view that the result of every measurement is determined by the statistical features of quantum mechanics arise because the

independently.

405

(2), the only features of (3) commonly used

$$\left. \begin{aligned} -1 \\ 0 \\ 1 \end{aligned} \right\} \quad (8)$$

probability distribution over all directions, and take

$$\rho(\lambda) = \delta(\lambda - \vec{a} \cdot \vec{a}) \quad (9)$$

then that \vec{a}' is obtained from \vec{a} by rotation towards \vec{b}

$$\langle A \rangle = P(\vec{a} \cdot \vec{b}) \quad (10)$$

in (8). For comparison, consider the replacement in the course of time by an ison-

$$\langle A \rangle = 1 - 2\theta/\pi \quad (11)$$

in (3), then (11) from (3).

in value -1 (at $\theta = 0$). It will be seen

the quantum mechanical correlation (3) if the

actively as well as on \vec{a} and \vec{b} . For ex-

ceeds \vec{b} until

$$\langle A \rangle \leq \vec{a} \cdot \vec{b} \quad (12)$$

values of the hidden variables, the results

$$\langle A \rangle \leq \vec{a} \cdot \vec{b} \quad (13)$$

normalized probability distribution,

$$\langle A \rangle = 1 - 2\theta/\pi \quad (14)$$

-1. It can reach -1 at $\vec{a} \cdot \vec{b} = \vec{b}$ only if

$$\langle A \rangle = 1 - 2\theta/\pi \quad (15)$$

(2) can be rewritten

$$\rho(\vec{a}, \lambda) \quad (16)$$

BELL

407

$$\rho(\vec{a}, \lambda) A(\vec{a}, \lambda) - A(\vec{a}, \lambda) \rho(\vec{c}, \lambda) \quad (17)$$

$$\rho(\vec{a}, \lambda) A(\vec{a}, \lambda) [A(\vec{b}, \lambda) A(\vec{c}, \lambda) - 1] \quad (18)$$

$$\rho(\lambda) [1 - A(\vec{a}, \lambda) A(\vec{c}, \lambda)] \quad (19)$$

$$\rho(\vec{b}, \lambda) - P(\vec{a} \cdot \vec{b}) \quad (20)$$

of order $|\vec{b} - \vec{c}|$ for small $|\vec{b} - \vec{c}|$. Thus $P(\vec{b}, \vec{c})$

and cannot equal the quantum mechanical

arbitrarily closely approximated by the form (2).

would not worry about failure of the approximation

(3) the functions

$$\frac{1}{2}(\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}) \quad (21)$$

\vec{a}' and $-\vec{a}' \cdot \vec{b}'$ over vectors \vec{a}' and \vec{b}' within spec-

and \vec{b} the difference is bounded by ϵ :

$$|\vec{a}' \cdot \vec{b}' - \vec{a} \cdot \vec{b}| \leq \epsilon \quad (22)$$

small.

$$\langle A \rangle \leq \vec{a} \cdot \vec{b} \quad (23)$$

$$\langle A \rangle \leq \vec{a} \cdot \vec{b} \quad (24)$$

$$\langle A \rangle \leq \vec{a} \cdot \vec{b} \quad (25)$$

$$\rho(\vec{a}, \lambda) \rho(\vec{b}, \lambda) \quad (26)$$

$$|\rho(\vec{b}, \lambda)| \leq 1 \quad (27)$$

$$\rho(\vec{a}, \lambda) + 1 \leq \vec{a} \cdot \vec{b} \quad (28)$$

$$\rho(\vec{a}, \lambda) \rho(\vec{b}, \lambda) - \rho(\vec{a}, \lambda) \rho(\vec{c}, \lambda) \quad (29)$$

$$\rho(\vec{b}, \lambda) \rho(\vec{c}, \lambda) [1 + \rho(\vec{a}, \lambda) \rho(\vec{c}, \lambda)] \quad (30)$$

$$\rho(\vec{c}, \lambda) [1 + \rho(\vec{a}, \lambda) \rho(\vec{b}, \lambda)] \quad (31)$$

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References

1. *Phys. Rev.* **47**, 777 (1935); see also N. BOHR, *Ibid.* **48**, 869 (1936), and D. R. INGLIS, *Rev. Mod. Phys.* **33**, 1 (1961).

2. This opinion, absolutely held fast: the real factual situation of the world with the system S_1 , which is spatially separated from S_2 , *Philosopher Scientist*, (Edited by P. A. SCHILP) p. 85, Open Court, Illinois (1949).

3. *Foundations of Quantum-mechanik*. Verlag Julius-Springer, Berlin (1955); J. M. JAUCH and C. PIRON, *Helv. Phys. Acta*, **28**, 1070 (1957).

4. *Foundations of Quantum-mechanik*, 2nd Ed., p. 37. The Clarendon Press, Oxford (1958).

5. *Foundations of Quantum-mechanik*, 2nd Ed., p. 37. The Clarendon Press, Oxford (1958).

6. *Foundations of Quantum-mechanik*, 2nd Ed., p. 37. The Clarendon Press, Oxford (1958).

represented, either accurately or arbitrarily. It requires little imagination to envisage the way, assuming [7] that any Hermitian operator result is easily extended to other systems. In this case we can always consider two dimensions, \vec{a} and \vec{b} , formally analogous to those of the subspace. Then for at least one quantum system, the statistical predictions of quantum mechanics to determine the results of individual measurements must be a mechanism whereby the settings of the instruments are made sufficiently by exchange of signals with velocity less than c of the type proposed by Bohm and Aharonov particles, are crucial. My useful discussions of this problem. The University; I am indebted to colleagues there for their helpful discussions.

Teorie skrytých parametrů

⊙ úplný popis reality

$[|st\rangle, \lambda]$

- kvantový stav

$|st\rangle$

- skryté parametry

λ

⊙ skryté parametry determinují výsledky všech měření

- nedeterminismus měření je dán neznalostí skrytých parametrů
- pravděpodobnosti KM průměrováním přes skryté parametry

Teorie skrytých parametrů

- ⊙ průměrování přes skryté parametry

$$\langle \text{st} | \hat{A} | \text{st} \rangle = \int d\lambda a(|\text{st}\rangle, \lambda)$$

$a(|\text{st}\rangle, \lambda)$ je jednoznačná hodnota veličiny v úplném stavu $[|\text{st}\rangle, \lambda]$

EPR systém

- ⊙ měření *korelace*

levého spinu ve směru \vec{a} a pravého spinu ve směru \vec{b}

- ⊙ kvantová předpověď

$$k_{\text{KM}}(\vec{a}, \vec{b}) = -\cos(\angle \vec{a}\vec{b}) = -\vec{a} \cdot \vec{b}$$

- ⊙ teorie skrytých parametrů

$$k_{\text{TSP}}(\vec{a}, \vec{b}) = \int d\lambda A(\vec{a}, \lambda) B(\vec{b}, \lambda)$$

EPR systém

⊙ teorie skrytých parametrů

$$k_{\text{TSP}}(\vec{a}, \vec{b}) = \int d\lambda A(\vec{a}, \lambda) B(\vec{b}, \lambda)$$

- $A(\vec{a}, \lambda) = \pm 1$ hodnota **levého spinu** ve směru \vec{a}
- $B(\vec{b}, \lambda) = \pm 1$ hodnota **pravého spinu** ve směru \vec{b}

EPR systém

⊙ teorie skrytých parametrů

$$k_{\text{TSP}}(\vec{a}, \vec{b}) = \int d\lambda A(\vec{a}, \lambda) B(\vec{b}, \lambda)$$

- $A(\vec{a}, \lambda) = \pm 1$ hodnota **levého spinu** ve směru \vec{a}
- $B(\vec{b}, \lambda) = \pm 1$ hodnota **pravého spinu** ve směru \vec{b}

⊙ předpoklad lokality

- $A(\vec{a}, \lambda)$ hodnota **levého spinu** nezávisí na **pravém směru** \vec{b}
- $B(\vec{b}, \lambda)$ hodnota **pravého spinu** nezávisí na **levém směru** \vec{a}

Bellovy nerovnosti

- antikorrelace pro stejné směry \Rightarrow

$$A(\vec{0}, \lambda) = -B(\vec{0}, \lambda)$$

- přímočará algebra \Rightarrow

$$1 + k_{\text{TSP}}(\vec{b}, \vec{c}) \geq \left| k_{\text{TSP}}(\vec{a}, \vec{b}) - k_{\text{TSP}}(\vec{a}, \vec{c}) \right|$$

IV. Contradiction

The main result will now be proved. Because ρ is a normalized probability distribution,

$$\int d\lambda \rho(\lambda) = 1, \quad (12)$$

and because of the properties (1), P in (2) cannot be less than -1 . It can reach -1 at $\vec{a} = \vec{b}$ only if

$$A(\vec{a}, \lambda) = -B(\vec{a}, \lambda) \quad (13)$$

except at a set of points λ of zero probability. Assuming this, (2) can be rewritten

$$P(\vec{a}, \vec{b}) = - \int d\lambda \rho(\lambda) A(\vec{a}, \lambda) A(\vec{b}, \lambda). \quad (14)$$

It follows that \vec{c} is another unit vector

$$\begin{aligned} P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c}) &= - \int d\lambda \rho(\lambda) [A(\vec{a}, \lambda) A(\vec{b}, \lambda) - A(\vec{a}, \lambda) A(\vec{c}, \lambda)] \\ &= \int d\lambda \rho(\lambda) A(\vec{a}, \lambda) A(\vec{b}, \lambda) [A(\vec{b}, \lambda) A(\vec{c}, \lambda) - 1] \end{aligned}$$

using (1), whence

$$|P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})| \leq \int d\lambda \rho(\lambda) [1 - A(\vec{b}, \lambda) A(\vec{c}, \lambda)]$$

The second term on the right is $P(\vec{b}, \vec{c})$, whence

$$1 + P(\vec{b}, \vec{c}) \geq |P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})| \quad (15)$$

Bellovy nerovnosti

- předpověď lokální teorie skrytých parametrů splňuje

$$1 + k_{\text{TSP}}(\vec{b}, \vec{c}) \geq \left| k_{\text{TSP}}(\vec{a}, \vec{b}) - k_{\text{TSP}}(\vec{a}, \vec{c}) \right|$$

- předpověď kvantové mechaniky nerovnost nesplňuje

$$1 - \vec{b} \cdot \vec{c} \geq \left| \vec{a} \cdot (\vec{b} - \vec{c}) \right| \quad \text{neplatí např. pro } (\vec{b} - \vec{c}) = \epsilon \vec{a}$$

$$\frac{1}{2}(\vec{b} - \vec{c})^2 \geq \epsilon |\vec{a} \cdot \vec{a}|$$

$$\frac{1}{2}\epsilon^2 \geq \epsilon$$

ale pro $\epsilon < 2$

spor

$$k_{\text{KM}}(\vec{a}, \vec{b}) = -\vec{a} \cdot \vec{b}$$

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

Bellovy nerovnosti

- ⊙ *kvantová mechanika není ekvivalentní lokální teorii skrytých parametrů*
- ⊙ *experimentální testy nerovností potvrzují předpověď kvantové mechaniky*

Distribuce hesla

dvě agentury distribující hesla (± 1) pro příchozí agenty

Alice

Bob

dvě organizace agentů

světlá strana

temná strana

do každé z agentur přijde agent, který prokáže svoji příslušnost

úloha

úkolem agentur je dát agentům ze světlé strany stejné heslo,
ve všech ostatních případech hesla různá

Distribuce hesla

úkolem agentur je dát agentům ze světlé strany stejné heslo,
ve všech ostatních případech hesla různá

klasické strategie distribuce hesel

lokální strategie může dosáhnout maximálně 75% úspěšnost

Distribuce hesla

úkolem agentur je dát agentům ze světlé strany stejné heslo,
ve všech ostatních případech hesla různá

příklad kvantové strategie distribuce hesel

Alice

Bob

$|EPR\rangle$

světlý → výsledek měření ↙

temný → výsledek měření ↗

světlá → výsledek měření →

temná → výsledek měření ↑

zaručuje ~85% úspěšnost distribuce hesel

Merminův–Peresův kouzelný čtverec

Alice

	+		
	-		
	-		
	-		

-

Alice dostane zadaný sloupec
má ho doplnit \pm tak aby součin byl -

Bob

+	-	-	+

+

Bob dostane zadaný řádek
má ho doplnit \pm tak aby součin byl +

úkolem je shodnout se v buňce na průsečíku

např.: P. K. Aravind, Am. J. Phys. 72 (2004) 1303

Merminův–Peresův kouzelný čtverec

Alice

Bob

řešení

dopředu si dohodnout předvyplněný čtverec splňující podmínky

součin v sloupci –

+	+	-	-
+	-	-	+
+	-	-	+
-	-	+	+

- - - -

součin v řádce +

+	+	-	-	+
+	-	-	+	+
+	-	-	+	+
-	-	+	+	+

úkolem je shodnout se v buňce na průsečíku

úspěšnost je s jistotou zaručena

Merminův–Peresův kouzelný čtverec

Alice

Bob

čtverec 3×3

dopředu si dohodnout předvyplněný čtverec splňující podmínky

součin v sloupci –

+	+	+
–	–	+
+	+	x

– – x

$x = ?$

součin v řádce +

+	+	+	+
–	–	+	+
+	+	x	x

úplná úspěšnost nelze zajistit

Merminův–Peresův kouzelný čtverec

Alice

$\hat{\sigma}_y \otimes \hat{1}$	$\hat{\sigma}_x \otimes \hat{1}$	$\hat{\sigma}_z \otimes \hat{1}$	$\hat{1} \otimes \hat{1}$
$\hat{\sigma}_z \otimes \hat{1}$	$\hat{\sigma}_x \otimes \hat{1}$	$\hat{\sigma}_y \otimes \hat{1}$	$\hat{1} \otimes \hat{1}$
$\hat{\sigma}_z \otimes \hat{1}$	$\hat{\sigma}_y \otimes \hat{1}$	$\hat{\sigma}_x \otimes \hat{1}$	$\hat{1} \otimes \hat{1}$
$\hat{\sigma}_x \otimes \hat{1}$	$\hat{\sigma}_y \otimes \hat{1}$	$\hat{\sigma}_z \otimes \hat{1}$	$\hat{1} \otimes \hat{1}$
$\hat{\sigma}_x \otimes \hat{1}$	$\hat{\sigma}_z \otimes \hat{1}$	$\hat{\sigma}_y \otimes \hat{1}$	$\hat{1} \otimes \hat{1}$
$\hat{\sigma}_y \otimes \hat{1}$	$\hat{\sigma}_z \otimes \hat{1}$	$\hat{\sigma}_x \otimes \hat{1}$	$\hat{1} \otimes \hat{1}$
$\hat{1} \otimes \hat{1}$	$\hat{1} \otimes \hat{1}$	$\hat{1} \otimes \hat{1}$	
$-\otimes$	$-\otimes$	$-\otimes$	
$\hat{1} \otimes \hat{1}$	$\hat{1} \otimes \hat{1}$	$\hat{1} \otimes \hat{1}$	

Bob

$\hat{1} \otimes \hat{\sigma}_y$	$\hat{1} \otimes \hat{\sigma}_x$	$\hat{1} \otimes \hat{\sigma}_z$	$\hat{1} \otimes \hat{1}$
$\hat{1} \otimes \hat{\sigma}_z$	$\hat{1} \otimes \hat{\sigma}_x$	$\hat{1} \otimes \hat{\sigma}_y$	$\hat{1} \otimes \hat{1}$
$\hat{1} \otimes \hat{\sigma}_z$	$\hat{1} \otimes \hat{\sigma}_y$	$\hat{1} \otimes \hat{\sigma}_x$	$\hat{1} \otimes \hat{1}$
$\hat{1} \otimes \hat{\sigma}_x$	$\hat{1} \otimes \hat{\sigma}_y$	$\hat{1} \otimes \hat{\sigma}_z$	$\hat{1} \otimes \hat{1}$
$\hat{1} \otimes \hat{\sigma}_x$	$\hat{1} \otimes \hat{\sigma}_z$	$\hat{1} \otimes \hat{\sigma}_y$	$\hat{1} \otimes \hat{1}$
$\hat{1} \otimes \hat{\sigma}_y$	$\hat{1} \otimes \hat{\sigma}_z$	$\hat{1} \otimes \hat{\sigma}_x$	$\hat{1} \otimes \hat{1}$
$\hat{1} \otimes \hat{1}$	$\hat{1} \otimes \hat{1}$	$\hat{1} \otimes \hat{1}$	
$-\otimes$	$-\otimes$	$-\otimes$	
$\hat{1} \otimes \hat{1}$	$\hat{1} \otimes \hat{1}$	$\hat{1} \otimes \hat{1}$	

Merminův–Peresův kouzelný čtverec

stav – dvojitý EPR stav

$$|S\rangle = \frac{1}{\sqrt{2}} \left(|EPR\rangle \otimes |EPR\rangle \right)$$

Alice a **Bob** provedou měření pozorovatelných v zadaných **sloupci** a **řádku**

(lze, protože pozorovatelné komutují)

výsledky vyplní do sloupce a řádku

(podmínky jsou splněny díky platnosti pro pozorovatelné)

na průsečíku se výsledek bude vždy shodovat

Merminův–Peresův kouzelný čtverec

3×3 hru lze díky kvantovým korelacím vyhrát vždy

opět

netrivialita kvantového chování